# AN INVESTIGATION OF THE HARD CONTRIBUTION TO $\phi$ PHOTOPRODUCTION

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**Abstract:** We investigate the possibility that the process of  $\phi$  photoproduction may have a significant hard perturbative QCD component. This suggestion is based on a study of the energy dependence of the forward  $\phi$  photoproduction cross section followed by a calculation where we show that a coherent sum of the pQCD and conventional soft Pomeron contributions provides an excellent reproduction of the experimental data. Our results suggest that the transition from the predominantly soft photoproduction of light  $\rho$  and  $\omega$  vector mesons to the predominantly hard photoproduction of heavy  $J/\Psi$  and  $\Upsilon$  is smooth and gradual, similar to the transition observed in deep inelastic scattering studies of the proton structure function in the small x limit. Our predictions for higher HERA energies are presented.

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#### 1 Introduction

Over the past few years we have seen a vigorous phenomenological investigation of the Pomeron through the study of hadronic total, elastic and diffractive cross sections, as well as the study of the proton deep inelastic scattering (DIS) structure function. In particular, Donnachie and Landshoff (DL) have promoted [1] an appealing and very simple Regge parameterization of the total hadronic cross sections in which

$$\sigma_{\text{tot}} = X \left(\frac{s}{s_0}\right)^{\Delta} + Y \left(\frac{s}{s_0}\right)^{-\eta}. \tag{1}$$

The two key ingredients of this approach are the Regge trajectories

$$\alpha_R(t) = \alpha_R(0) + \alpha_R' t, \qquad (2)$$

where  $\alpha_R(0) = 1 - \eta$  and the Pomeron trajectory, which dominates at high energies,

$$\alpha_P(t) = \alpha_P(0) + \alpha_P' t, \qquad (3)$$

where  $\alpha_P(0) = 1 + \Delta$ . The DL study establishes universal values  $\Delta = 0.0808$  and  $\eta = 0.4525$ . This study is supplemented by the analysis of Block, Kang and White [2] who determine the slope  $\alpha_P'$  of the Pomeron trajectory to be  $\alpha_P' = 0.2 \,\text{GeV}^{-2}$ .

The same approach is also applicable to the analysis of real photoproduction and of the proton DIS structure function [3]. While the energy dependence of the photoproduction total cross section follows the DL pattern, it has been observed [4] that  $F_2(x, Q^2)$  behaves, for small enough x, like  $x^{-\lambda}$ , where  $\lambda$  is slowly growing with  $Q^2$ . The growth of  $\lambda$  is associated with the behavior of the gluon structure function in the small x limit

$$\lambda = \frac{\partial \ln (xG(x,Q^2))}{\partial \ln(1/x)}.$$
 (4)

It has been recognized for quite some time that the transition from the predominantly soft real photoproduction  $(Q^2 = 0)$  to the predominantly hard DIS processes, with high enough  $Q^2$ , is smooth and gradual [3, 5, 6, 7]. This observation, regardless of its theoretical interpretation, is evident once we examine the energy dependence of  $F_2(x, Q^2)$  in the small x limit with  $Q^2$  ranging from zero to a few GeV<sup>2</sup>. It is also well known that real photoproduction of light vector mesons,  $\rho$  and  $\omega$ , is dominated by a soft Pomeron exchange [8][9], whereas photoproduction of heavy vectors,  $J/\Psi$  and  $\Upsilon$ , is well reproduced by a perturbative QCD (pQCD) calculation [10, 11, 12, 13], where  $M_V^2/4$  replaces  $Q^2$  as a measure of the process hardness. Although  $M_V^2/4$  is a discrete variable, while  $Q^2$  is a continuous DIS variable, it

is interesting to check if the transition from soft to hard photoproduction of vector mesons follows the behavior pattern observed in DIS. To this end, the study of  $\phi$  photoproduction is particularly instructive, as  $M_{\phi}^2/4 = 0.26 \,\mathrm{GeV}^2$ , while we know that the energy dependence of  $F_2$ , with small x and  $Q^2$  as low as  $0.2 - 0.3 \,\mathrm{GeV}^2$ , is steeper than the energy dependence of  $\sigma_{\mathrm{tot}}(\gamma p)$ .

Our investigation is susceptible to both experimental and theoretical uncertainties. Experimentally, a systematic study of the integrated  $\phi$  photoproduction cross section and the forward differential cross section slope [14, 15, 16, 17] are not very reliable as different experimental groups have utilized different, not always mutually consistent, methods to extract and relate these quantities. To overcome this difficulty, we have analyzed the measured differential cross sections rather than integrated quantities. Even so, the two higher energy data points [16, 17] are averaged over wide energy bins. This, combined with the overall poor quality of the reported data, may make a detailed analysis non conclusive at this stage. Theoretically, since we wish to utilize the gluon structure function at low  $Q^2$ , the calculation of the hard component requires some clarifications. Technically, a pQCD calculation of  $[d\sigma(\gamma p \to \phi p)/dt]_{t=0}$  depends on our knowledge of the gluon structure function at  $Q^2 = 0.26 \,\mathrm{GeV}^2$ . Such information requires an extrapolation of a given parton distribution below its initial evolution threshold  $Q_0^2$ . For this purpose we adopt a linear extrapolation which was successfully utilized in previous calculations [6, 18]. As we shall see, there is a significant difference between the MRST [19] and GRV98 [20] input gluon distributions. We have chosen to use the GRV98 distribution and shall explain our motivations for doing so.

The purpose of this letter is to examine these issues in some detail from different points of view. We present an analysis of the existing  $\phi$  photoproduction forward differential cross section data which suggests an energy dependence which is steeper than the typical energy dependence associated with the soft Pomeron [1]. We then present a pQCD calculation from which we deduce that the hard component is responsible for about a quarter of the  $\phi$  photoproduction amplitude in the forward direction at presently available energies. We then proceed to show that a coherent sum of the calculated pQCD amplitude and a conventional soft Pomeron exchange contribution provides an excellent reproduction of the available data [14, 15, 16, 17].

# 2 Data analysis

Our data analysis investigates whether the  $\phi$  photoproduction cross section follows a power dependence on the c.m. energy W, and whether this power is larger than the value determined from the energy dependence of the total cross section. Following DL [1] and Block et

al. [2], we expect the  $\phi$  photoproduction cross section to behave like  $W^{4\Delta}$  and the forward slope to behave like  $4\alpha'_P \ln W$ . The analysis of  $\phi$  photoproduction data is seemingly easy, as this process proceeds exclusively through Pomeron exchange, since the various Regge exchanges cancel each other. The problem is that the published analysis [1] [17] depends on a comparison between integrated cross section data taken by different groups who have utilized different, and not always mutually consistent, procedures. In addition, because the  $\phi$  forward slope is shrinking, the interpretation of the integrated cross section behaving as a fixed power of W is somewhat ambiguous. In order to bypass these difficulties, we have limited ourselves to the analysis of the individual differential cross sections  $d\sigma/dt$  as reported by the experimental groups [14, 15, 16, 17]. We have used data with W > 6 GeV corresponding to x < 0.025.

Fig. 1 shows that  $(d\sigma/dt)_0$  in the available energy range is, indeed, well fitted by an effective power of W,

$$\left(\frac{d\sigma(\gamma p \to \phi p)}{dt}\right)_{t=0} = A W^{4\lambda}.$$
(5)

Our best fit, for 5 data points with  $6.7 \le W \le 70\,\mathrm{GeV}$ , has an excellent  $\chi^2/n.d.f. = 0.22$ , corresponding to  $A = 0.76 \pm 0.09$  and  $\lambda = 0.135 \pm 0.012$ . For comparison we show also a fit where we fix the power to its DL value  $4\Delta = 0.3232$  and obtain  $A = 1.21 \pm 0.06$  with  $\chi^2/n.d.f. = 0.92$ . This is a lesser quality fit, but it cannot be discredited. Our inability to determine the power unambiguously results from the big error coupled to the ZEUS high energy data point [17]. Even so, it is clear that an improvement of the HERA data point at  $< W >= 70\,\mathrm{GeV}$  and relevant data at higher energies will enable us to conclusively distinguish between a DL type interpretation and ours.

In order to further examine the suggestion that the dependence of  $\phi$  photoproduction on W is steeper than the behavior implied by a soft Pomeron exchange, we have studied two (related) ratios

$$R_1 = \frac{(d\sigma/dt)_0}{\sigma_{\text{tot}}^2(\phi p)} \,, \tag{6}$$

and

$$R_2 = \frac{(d\sigma/dt)_0}{\sigma_P^2(\gamma p)} \,, \tag{7}$$

where  $\sigma_P(\gamma p)$  is the soft Pomeron exchange contribution to  $\sigma_{\text{tot}}(\gamma p)$ . Both ratios are shown in Fig. 2. A careful study of these ratios is of interest as both  $\sigma_P^2(\gamma p)$  and  $\sigma_{\text{tot}}^2(\phi p)$  behave as  $W^{4\Delta}$ . Using the DL parameterization [1] we have

$$\sigma_P(\gamma p) = 67.7 \left(\frac{W}{W_0}\right)^{0.1616} \mu b$$
 (8)

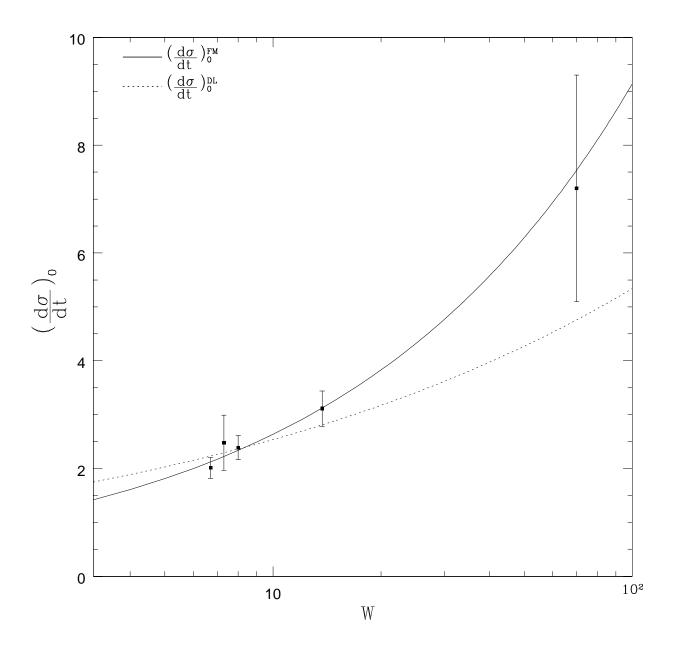


Figure 1: Best power fit compared with DL prediction for  $[d\sigma(\gamma p \to \phi p)/dt]_{t=0}$ 

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and, with aid of the additive quark model relation between  $\phi p$ , Kp and  $\pi p$  cross sections,

$$\sigma_{\text{tot}}(\phi p) = 10.01 \left(\frac{W}{W_0}\right)^{0.1616} + 1.51 \left(\frac{W}{W_0}\right)^{-0.4525} \text{ mb},$$
 (9)

with  $W_0 = 1 \,\text{GeV}$ . A predominantly soft production mechanism for  $\gamma p \to \phi p$  means that both ratios presented in Fig. 2 are constants. The best fits that we have obtained imply that  $R_1$  behaves as  $W^{0.278\pm0.086}$  and  $R_2$  behaves as  $W^{0.215\pm0.020}$ . Constant ratios provide marginally acceptable fits to  $R_1$  and  $R_2$ , but this option cannot be definitely excluded.

To summarize: The available experimental data on  $(d\sigma(\gamma p \to \phi p)/dt)_0$  is consistently well described as having an energy dependence steeper than the one implied by a soft Pomeron exchange. Nevertheless, the assumption of a pure soft production mechanism cannot be unambiguously eliminated.

## 3 A pQCD calculation

Our pQCD calculation of the forward  $\phi$  photoproduction follows earlier pQCD calculations of the forward photoproduction cross section of heavy vector mesons [10, 11, 12, 13]. These calculations are considerably simplified once we assume a non-relativistic wave function for those vector meson states. This assumption, which is also valid for  $\phi$ , enables us to write a leading-order expression

$$\left[\frac{d\sigma(\gamma p \to \phi p)}{dt}\right]_{t=0} = \frac{\alpha_S^2 \Gamma_{ee}^{\phi}}{3\alpha_{EM} M_{\phi}^5} 16\pi^3 \left[xG\left(x, \frac{M_{\phi}^2}{4}\right)\right]^2,\tag{10}$$

where  $x = (M_{\phi}/W)^2$  and  $\Gamma_{ee}^{\phi}$  is the partial decay width of  $\phi \to e^+e^-$ . In the following we follow Refs. [12][13] and, after calculating the cross section resulting from the imaginary forward amplitude in leading order, we correct for the real part of the amplitude and for higher orders.

The basic problem with our suggested calculation is that we depend on the knowledge of the gluon structure function  $xG(x,Q^2)$  at  $Q^2 = M_{\phi}^2/4 = 0.26 \,\text{GeV}^2$ . This  $Q^2$  value is well below  $Q_0^2$ , the initial evolution threshold, used in the updated parton distribution parameterizations [19][20] which took into account the behavior of  $F_2$  and its logarithmic derivatives at small  $Q^2$ . We recall the general property of the gluon structure function which is linear in  $Q^2$  in the limit of very small virtuality and use, accordingly, a linear extrapolation

$$xG(x,Q^2) = \frac{Q^2}{Q_0^2} xG(x,Q_0^2) , Q^2 < Q_0^2 .$$
 (11)

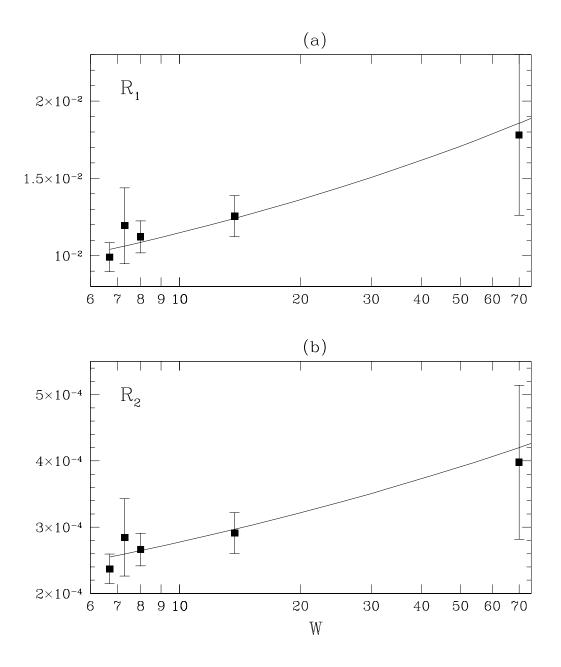


Figure 2:  $R_1$  and  $R_2$  data and best fits.

This approximation has been successfully used in previous theoretical DIS investigations [6][18] in which the knowledge of xG in the small  $Q^2$  region was required.

The results of our gluon structure function extrapolation for MRST [19] and GRV98 [20], at the relevant  $Q^2 = 0.26 \,\text{GeV}^2$  value, are shown in Fig. 3. Note that  $Q_0^2 = 1.2 \,\text{GeV}^2$  for MRST and  $0.8 \,\text{GeV}^2$  for GRV98. Fig. 3 shows also the Pomeron term of ALLM97 [3]. In the following we have used the GRV98 extrapolated distribution. Our motivation is double folded:

- 1) From a practical point of view, the MRST gluon structure function is considerably smaller than GRV98. If we adopt the MRST distribution, we get a diminishing small hard contribution for  $\phi$  photoproduction in the W range of interest. If our data analysis is substantiated, we would then be left with the need for some explanation for the observed energy dependence of the  $\phi$  data.
- 2) Theoretically, we note that MRST is very close to ALLM97 in the x region of interest. Our interpretation is that the MRST input at small  $Q^2$  is predominantly soft, such as is ALLM97, and thus, even though perfectly legitimate, it is less suitable for our analysis.

We follow Ref.[12, 13] and consider the following corrections to the leading order cross section written in Eq. (10):

- 1) Eq. (10) corresponds to the imaginary part of the forward amplitude and should be corrected by  $(1 + \rho^2)$ , where  $\rho = \text{Re}A/\text{Im}A$ , and A is the amplitude under consideration. We recall that  $\rho \simeq \pi \lambda/2$ , where, in our case,  $\lambda$  is given by Eq. (4). We note that  $\lambda = 0.145$  for  $W = 70\,\text{GeV}$  and decreases very slowly as the energy decreases.
- 2) The next to leading order corrections are estimated by  $[1 + 0.5\alpha_S(M_\phi^2/4)]$ .
- 3) Relativistic corrections and the effects of intermediate off diagonal partons in  $\phi$  photoproduction are rather small and have been neglected.

With these effects put together, our overall correction factor is C=1.21 at the high energy end of our data, decreasing slowly to 1.16 in the lowest energy. The energy dependence of our calculated hard cross section is given in Table 1 and displayed in Fig. 4.

## 4 A hybrid pQCD and soft Pomeron model

Although the pQCD contributions calculated in the previous section provide a significant contribution to  $\phi$  photoproduction, there is no doubt that, in the energy range under consideration, the leading production mechanism is a soft Pomeron exchange. Accordingly, we

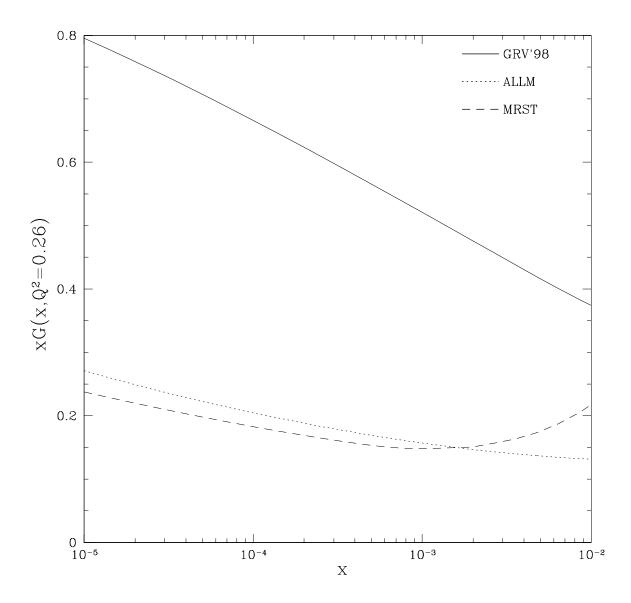


Figure 3: ALLM97 and the extrapolated MRST and GRV98 parameterizations for the gluon structure function  $xG(x,Q^2)$  at  $Q^2=0.26\,\mathrm{GeV}^2$ .

| W    | $\left(\frac{d\sigma}{dt}\right)_0^{\text{exp}}$ | $\left(\frac{d\sigma}{dt}\right)_0^{\text{FM}}$ | $\left(\frac{d\sigma}{dt}\right)_0^{\text{soft}}$ | $\left(\frac{d\sigma}{dt}\right)_0^{\text{pQCD}}$ |
|------|--|---|---|---|
| 6.7  | $2.01 \pm 0.19$                                  | 2.20  | 1.28  | 0.13  |
| 7.3  | $2.48 \pm 0.51$                                  | 2.26  | 1.30  | 0.13  |
| 8.0  | $2.39 \pm 0.22$                                  | 2.35  | 1.34  | 0.14  |
| 13.7 | $3.11 \pm 0.33$                                  | 2.91  | 1.60  | 0.20  |
| 30   |  | 3.96  | 2.06  | 0.31  |
| 50   |  | 4.79  | 2.43  | 0.40  |
| 70   | $7.20 \pm 2.10$                                  | 5.40  | 2.71  | 0.46  |
| 100  |  | 6.10  | 3.04  | 0.53  |
| 150  |  | 6.97  | 3.46  | 0.61  |
| 200  |  | 7.65  | 3.80  | 0.67  |
| 250  |  | 8.20  | 4.08  | 0.71  |

Table 1:  $(d\sigma/dt)_{t=0}$  data and calculations.

attempt to fit the data with a simple hybrid two- component model with the following prescriptions:

1) The first component is a soft DL Pomeron with an  $\alpha_P(t)$  intercept of 0.0808, namely

$$\left(\frac{d\sigma}{dt}\right)_{t=0}^{\text{Soft}} = A_S^2 \left(\frac{W}{W_0}\right)^{0.3232}.$$
(12)

- 2) A hard pQCD component  $\left(\frac{d\sigma}{dt}\right)_{t=0}^{\text{Hard}}$ , as calculated in the previous section.
- 3) A Coherent sum of the two component amplitudes.

We fit the 5 data points of  $(d\sigma/dt)_0$  with one parameter, the normalization  $A_S$  of the DL Pomeron. We obtain a best fit value  $A_S = [0.83 \pm 0.02](\mu b)^{0.5}$  with  $\chi^2/n.d.f. = 0.54$ . Our fitted cross sections (called FM) are presented in Table 1 and displayed in Fig. 4, both table and figure showing also the pQCD and soft cross sections and our predictions for higher HERA energies. Our fit should be compared with the power fit, presented earlier, which has a  $\chi^2/n.d.f. = 0.22$  and the conventional DL fit which has  $\chi^2/n.d.f. = 0.92$ . Clearly, the presently available data is not sufficient to rule out any of these options. Once again we note that this ambiguity results from the big error associated to the  $\langle W \rangle = 70 \,\text{GeV}$  point. Improvement of the quality of this point and additional HERA data will enable a more discriminative analysis.

The hybrid model that we have just suggested can be further examined by considering the differential cross sections. Such data is available [16, 17] at < W > = 13.7 and 70 GeV.

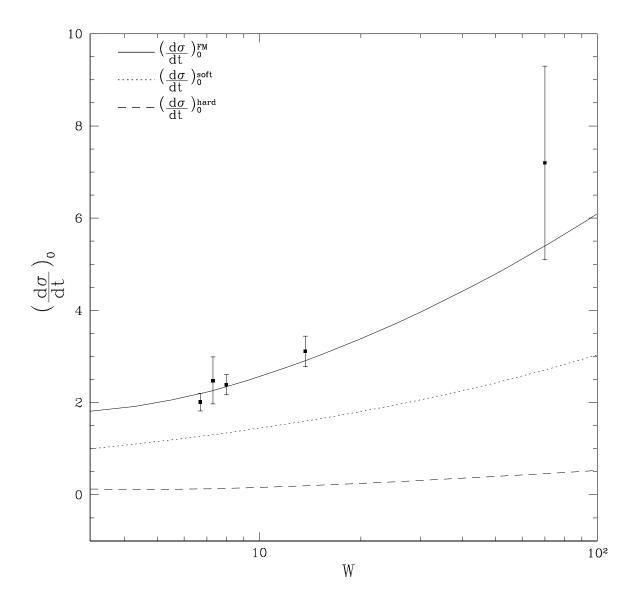


Figure 4: Data and our calculation for  $(d\sigma/dt)_{t=0}$ . Also shown are our separate calculations of the soft and hard cross sections.

For the purpose of our calculations we need to know the t-dependence of both the pQCD and soft Pomeron amplitudes. To this end we assume each of these dependences to be approximated by an exponential,

$$F_i = F_i(0)e^{\frac{B_i}{2}t} \quad i = S, H.$$
 (13)

This simple approximation is sufficient here, considering the quality of the available data on  $\phi$  photoproduction. We then proceed as follows:

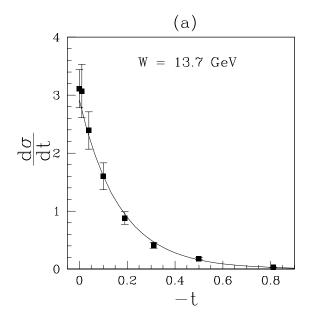
- 1) We take for the pQCD amplitude an energy independent exponential slope. We derive its value from the high energy differential cross section on  $J/\Psi$  photoproduction combined with the observation [21] that this slope is energy independent, corresponding to a flat hard Pomeron. In our analysis we have taken  $B_H = 4.6 \,\text{GeV}^{-2}$ , which corresponds to the H1 measurement [22]. An equally good fit is obtained also with the ZEUS value [23] of  $B_H = 4.0 \,\text{GeV}^{-2}$ . We have also treated  $B_H$  as a free parameter.
- 2) For the soft Pomeron amplitude we assume a conventional Regge type exponential slope depending on two fitted parameters,

$$B_S = B_0^S + 2\alpha_P' \ln\left(\frac{W}{W_0}\right)^2. \tag{14}$$

The  $B_0^S$  approximate value is known from the phenomenology of  $\phi$  photoproduction at low energies and  $\alpha'_P$  is approximately known from high energy hadron-hadron phenomenology [2]. As we show below, the fitted values of these parameters are in excellent agreement with our expectations.

This hybrid model was fitted to reproduce  $13 \ (d\sigma/dt)$  data points measured at < W > = 13.7 and  $70 \,\text{GeV}$ . We obtain  $B_0^S = 4.20 \pm 0.90 \,\text{GeV}^{-2}$  and  $\alpha_P' = 0.20 \pm 0.08$  with  $\chi^2/n.d.f. = 0.65$ . Our results are in an excellent agreement with soft hadron Regge analysis [2]. Our soft exponential slopes extrapolate well into the predominantly soft low energy experimental data on  $\phi$  phtoproduction [24]. The data and our fit are shown in Fig. 5. A slight improvement of the fit is obtained if we take  $B_H$  as a free parameter and obtain  $B_H = 3.50 \pm 0.60 \,\text{GeV}^{-2}$ , with the other parameters changing insignificantly.

Although our  $(d\sigma/dt)$  fit corroborates our proposition that high energy  $\phi$  real photoproduction has a significant hard component, we do not consider our success to be decisive. The  $< W >= 70\,\mathrm{GeV}$  data, on its own, can be equally well described by a single DL type soft component. Our two component model is appropriate to describe the  $< W >= 13.7\,\mathrm{GeV}$  data including  $t > 0.4\,\mathrm{GeV^2}$ , where a single exponent is not sufficient. However, we caution against reaching too strong a conclusion from a single measurement. Clearly, and not only for the purpose of our analysis, additional knowledge on higher t behavior will help to clarify the picture.



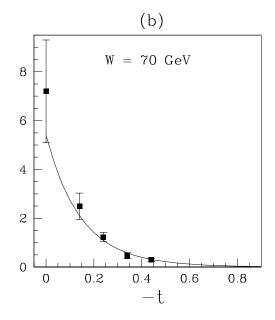


Figure 5: Data and our calculations for  $(d\sigma/dt)$  at < W> = 13.7 and 70 GeV.

#### 5 Discussion

Following are the main conclusions of our study and some general remarks:

- 1) The data analysis of forward real  $\phi$  photoproduction suggests the existence of a significant hard pQCD contribution in the energy range of  $6 \le W \le 70 \,\text{GeV}$ .
- 2) The above suggestion is corroborated by a pQCD calculation of  $[d\sigma(\gamma p \to \phi p)/dt]_0$  using the GRV98 gluon distribution extrapolated to  $Q^2 = 0.26 \,\text{GeV}^2$ .
- 3) A hybrid model, in which we coherently add the parameter free pQCD hard component and a DL type soft component, provides an excellent overall reproduction of the data. We note that the ratio of hard to soft contribution in our model is compatible with the ratio obtained in a detailed study [6] of DIS in comparable small  $Q^2$  values.
- 4) Our hybrid model is significantly different from the two Pomeron model suggested in Ref. [7]. In our model the t = 0 intercept of the hard effective Pomeron is given by Eq. (4) and as such it is a moving pole. In the model of Ref. [7], the hard Pomeron is a fixed pole with a comparatively high  $\lambda$ .
- 5) Theoretically, the validity of our calculation rests on (i) the legitimacy of our  $Q^2$  extrapolation of  $xG(x,Q^2)$  below  $Q_0^2$ ; and (ii) our choice of GRV98 for the input gluon structure function.
- 6) Our overall analysis strongly supports the existence of a significant hard component contributing to  $\gamma p \to \phi p$ . However, a decisive quantitative conclusion depends on improving and extending the HERA data on  $\phi$  photoproduction.

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